Spreading of damage in the ballistic deposition and larger curvature models

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(Received 31 January 1996)

The damage spreading of various growth models is described. The damage spreading distance D of an initial small perturbation grows as t^{γ} with time t. In the ballistic deposition model and the restricted solid-on-solid growth model γ is consistent with $1/z$ implying that D is proportional to the parallel correlation length obtained from the usual surface scaling where z is the dynamic critical exponent. The survival probability of an initial perturbation decays with a power law as a function of time. For the larger curvature model, however, the damage spreading distance grows much faster than the parallel correlation length. Possible implications of the damage spreading idea to the Family model are discussed. [S1063-651X(96)10710-8]

PACS number(s): 05.70.Ln, 68.55.−a, 68.35.Fx, 05.40.+j

The study of surface fluctuations on various growth models has become a very attractive area recently [1]. It is related to the surface growth of a thin film on the vacuum deposition such as the molecular beam epitaxy, where a beam of particles is normally incident on a flat substrate, and the random stochastic noise is present in the impinging flux [2]. Since the surface width of the nonequilibrium growth process follows a simple scaling form, most efforts have concentrated on measuring the surface fluctuations. The surface width W is defined as the standard deviation of the surface height. In a finite system of lateral size L, the width W starting from a flat substrate scales as [3]

$$W(t)\sim\xi^{\alpha}. \quad (1)$$

ξ is the correlation length parallel to the substrate following

$$\begin{aligned}\xi(t)&\sim Lf(t/L^{z})\\ &\sim t^{1/z},\quad t\ll L^{z}\\ &\sim L,\quad t\gg L^{z},\end{aligned} \quad (2)$$

where the scaling function $f(x)$ is $x^{1/z}$ for $x\ll 1$ and is constant for $x\gg 1$. The correlation length denotes how the surface height correlations spread over the substrate. Combining Eqs. (1) and (2) one obtains the scaling behavior of W as [3]

$$\begin{aligned}W(t)&\sim\xi^{\alpha}\\ &\sim t^{\beta},\quad t\ll L^{z}\\ &\sim L^{\alpha},\quad t\gg L^{z},\end{aligned} \quad (3)$$

where the exponents β and z are connected by the relation $z\beta=\alpha$.

On the other hand, much attention has focused on the concept of damage spreading [4] in the Ising model recently because the damage spreading is a possible method for obtaining correlation function. The interesting quantity is the damage spreading distance D which is the propagation distance of an initial small perturbation at $t=0$. Here, we apply the damage spreading idea to various discrete growth problems and we investigate the relation between the correlation length of the surface height and the propagation distance of the perturbation. Our numerical simulation shows that D is proportional to $t^{1/z}$ for the ballistic deposition (BD) model [5], the restricted solid-on-solid (RSOS) model [6], and the Family model [7]. However, in the larger curvature (LC) model [8], D is larger than $t^{1/z}$ implying that the initial perturbation propagates faster than the correlation length.

We consider two systems A and B and start from two different initial conditions, which are the same except one point at r_0. For example, the initial condition in system A is flat,

$$h^{A}(r,0)=0, \quad (4)$$

for all position r, where $h(r,t)$ represents the surface height at time t. In system B, the initial condition is

$$\begin{aligned}h^{B}(r,0)&=0\ \text{ for } r\neq r_0\\ &=n\ \text{ for } r=r_0,\end{aligned} \quad (5)$$

where n is a small integer. In most of our simulation, we choose $n=+1$. The only difference in the initial conditions is that the system B has a small bump at r_0. We allow the surfaces in the two systems to evolve under *the same growth rules and under the same sequence of random numbers*. Then, the surface configurations of them evolve differently due to the different initial conditions. A damage site is defined as the point where the surface heights $h^{A}(r,t)$ and $h^{B}(r,t)$ are not the same. Since we are interested in the propagation of the initial perturbation, we define the damage spreading distance (or propagation distance) D as the maximum value among the distances between the damage sites and the original point r_0,

$$D=\max\{|r_i-r_0|\}, \quad (6)$$

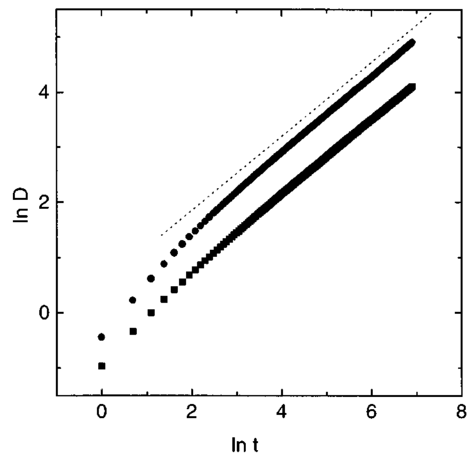

FIG. 1. The damage spreading distance D as a function of time in log-log plot for the RSOS model (bottom) and the BD model (top) with $n=1$. The dotted line is a guide line for $\gamma=2/3$.

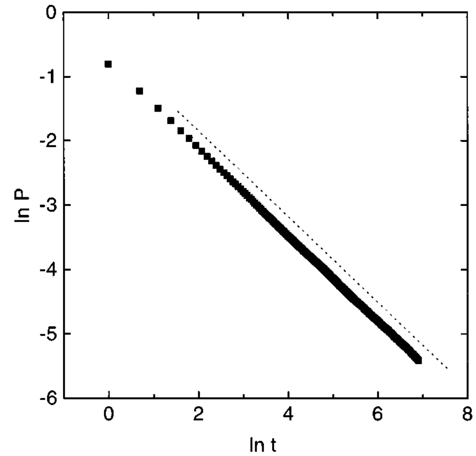

FIG. 2. The survival probability $P(t)$ as a function of time in log-log plot for the RSOS model.

where max takes the maximum for any r_i in the damage sites.

We consider four different growth models well known in the literature [1]. For completeness, here we describe the models briefly. The general growth algorithm is to select a site randomly on a $(d-1)$-dimensional substrate and the next steps depend on the models. (a) The RSOS model: allow growth by one provided the nearest neighbor height difference is not larger than one in the configuration [6]. (b) The BD model: allow a particle to fall along a straight line perpendicular to the substrate, until it sticks to the nearest site of the particle on the line or to the top of the column [5]. (c) The Family model [7] (the LC model [8]): the dropped particle can migrate to the lower height site (the larger curvature site) among the nearest neighbor sites.

It is generally believed that these discrete growth models are described by a continuum equation [9–12]

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = \nu_2 \nabla^2 h + \lambda (\nabla h)^2 - \nu_4 \nabla^4 h + \eta(\mathbf{x},t), \tag{7}$$

where $\eta(\mathbf{x},t)$ is a *nonconserved*, uncorrelated random noise. In the Family model and the LC model, the growth process is conservative meaning that the growth process conserves the total particles after deposition, which implies that evaporation and the formation of void and overhangs are negligible. Such a conserved growth process can be derived by a conserved current equation $(\lambda=0)$,

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = -\nabla \cdot \mathbf{j}(\mathbf{x},t) + \eta(\mathbf{x},t) \tag{8}$$

with the surface current $\mathbf{j}$,

$$\mathbf{j}(\mathbf{x},t) = -\nu_2 \nabla h + \nu_4 \nabla^3 h. \tag{9}$$

For $\nu_4=0$ and $\lambda=0$, it belongs to the Edwards and Wilkinson (EW) diffusion equation [10] with $\alpha=(3-d)/2$ and $z=2$ on $d-1$ substrate dimension. The Family model follows the EW universality class. When $\nu_2=0$ and $\lambda=0$, it is Mullin's [11] linear equation, giving $\alpha=(5-d)/2$ and $z=4$, i.e., $\beta=(5-d)/8$. The LC model is a simple curvature driven model described by Mullin's equation [8]. If $\lambda\neq 0$, the equation cannot be written in the form of Eq. (8) and it is a nonconserved growth equation. When $\nu_4=0$, the equation becomes the well-studied Kardar-Parisi-Zhang (KPZ) equation [9]. Both the RSOS model and the BD model belong to the KPZ universality class, where z is 3/2 in $d=1+1$.

Following the growth rules, we deposit particles on the two systems A and B which have the different initial conditions given in Eqs. (4) and (5) with $n=1$. We apply the periodic boundary condition and use the same sequence of random numbers for both systems. So the dropped particles are deposited on the same position in both A and B systems provided that the local height configurations of them are the same. At $t=0$, the only damage site is at r_0, and then the damage site can diffuse, annihilate itself, or create another damage site on the nearest neighbors. If there exists any damage sites at t, we call it an *''active state''* at that time. If all the damage sites disappear at time t_c, it becomes a *''dead state.''* The surface configurations of the two systems become identical and they evolve the same way after t_c. There is no further creation of a damage site from the dead state. Hence we classify the surface configurations as either a dead state or an active state. If the system falls into a dead state we stop the run and start another run. The damage distance as a function of time is measured by averaging over the active states only.

We have simulated the models on a one-dimensional substrate of lateral size $L=1000$ with the given initial conditions. All simulation results are averaged over 1000 independent runs. The natural time t is defined as the number of average Monte Carlo steps per site. The maximum time is chosen as 1000 to maintain the condition that $D \ll L$. In this way the damage spreading distance has no finite system size effect. By definition $D=0$ at $t=0$ and we find that D grows with time following a power law:

$$D(t) \sim t^{\gamma}. \tag{10}$$

The time evolutions of the damage distances averaged over the active states for both the BD model and the RSOS model are shown in Fig. 1. We get very nice power law behaviors

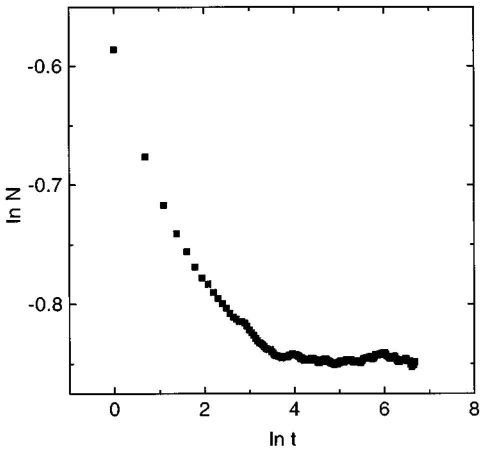

FIG. 3. The average number of damage sites $N(t)$ as a function of time in log-log plot for the RSOS model.

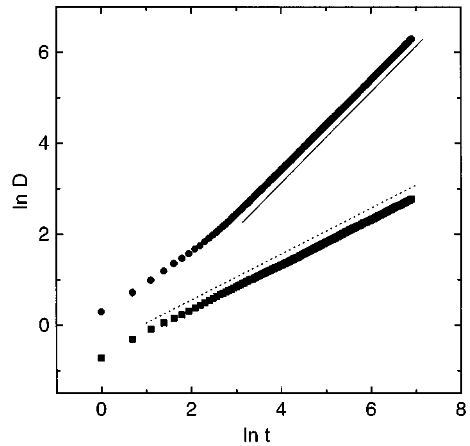

FIG. 4. The damage spreading distance D as a function of time in log-log plot for the Family (bottom) model and the LC (top) model. The slope $(=\gamma)$ is around 0.5 for the Family model. However the slope increases with time and approaches one in the LC model. The solid line is for $\gamma=1$ and the dotted line is for $\gamma=1/2$.

of D following Eq. (10). From the log-log plot of the damage distance against time we obtain the value of the exponent γ,

$$\gamma=0.67\pm0.01, \quad d=1+1 \tag{11}$$

for the both models. This γ is in a good agreement with the relation

$$\gamma=1/z \tag{12}$$

with the value $z=3/2$ of the KPZ class in $d=1+1$. Another initial condition with $n=-1$ produces the same behavior quantitatively. So D is proportional to the correlation length ξ.

Since the models are described by the nonconserved current equation, a dead state can be developed with time. We define the survival probability $P(t)$ as

$$P(t)=\frac{[\text{number of active states } (t)]}{[\text{number of total states } (t)]} \tag{13}$$

at time t, where the number of total states is the sum of both the active states and the dead states. Figure 2 shows the power law dependence of the survival probability, $P(t)\sim t^{-\delta}$ for the RSOS model with $\delta\approx2/3$. The value of δ for the BD model remains the same. This power law behavior of the survival probability is similar to that of the continuous absorbing transition [13]. The damage sites are linked to each other and they form one connected cluster. We also define $N(t)$ as the average number of the damage site over all the states [14]. Then, $N(t)$ should be proportional to $D(t)\times P(t)$. Since the value of δ is very close to γ, $N(t)$ remains almost constant after the initial transient regime $(t>40)$ as shown in Fig. 3.

In the conserved surface current models such as the LC model or the Family model, the deposited particles remain on the surface without overhangs or vacancies. Since the total number of particles after deposition are conserved, the number of particles in the system B is always larger than that of the system A up to n. Therefore there are no dead states in the conserved growth model. In the Family model, we get $\gamma\approx0.50\pm0.01$ as shown in Fig. 4, again supporting the relation $\gamma=1/z$ with $z=2$. Surprisingly, there is only one damage site at all the time and the damage site (one more extra particle in the system B when $n=1$) behaves like a random walk. The same behavior was observed even in $d=2+1$. This supports that the Family model has $z=2$ independent of the dimension. However, in the LC model, the number of the damage sites is not always one. Actually it increases with time. In contrast to the RSOS model, the damage sites are not always connected to each other and they can form many separated clusters. The averaged damage spreading distance is shown in Fig. 4 for the LC model. At the beginning the slope is around 0.67, then it keeps on growing and approaches one. The damage distance increases almost linearly in time after the initial transient regime. Due to the growth rules, the upper bound of the γ is one in general. In the LC model, $z=4$ is well known and the correlation length grows as $t^{1/4}$ [8]. The initial perturbation propagates much faster than the correlation length so that the propagation length of the perturbation is not related to the correlation length in the LC model. The similar behavior was observed in the Wolf and Villian (WV) model [15]. In the Family, BD, and RSOS models, the growth rules depend on the height configuration of the nearest neighbors. However, in the LC model the growth algorithm depends on the height configurations of not only the nearest neighbors but also the next nearest neighbors. The damage propagation distance is a kind of the response distance of the initial perturbation. The response function is related to the correlation function by the fluctuation dissipation theorem. A possible speculation on the reason why the damage propagates faster than the correlation length in the LC model is that the damage spreading distance may be related to the small wavelength response function due to the small initial perturbation. In the RSOS model the damaged sites form one big cluster producing long wavelength response, but in the LC model the damaged sites do not make a big connected cluster. So the damage spreading method may not always estimate the correlation length.

In summary, we have applied the damage spreading idea to various growth models and have measured the propagation distance of a small initial perturbation as a function of time. The damage spreading method can separate the influence of the initial perturbation on the subsequent surface growth. The measured damage spreading distance is proportional to the parallel correlation length obtained from the scaling form in both the Family model (EW class) and the RSOS model (KPZ class). So the damage spreading distance in the dynamic growth models can be interpreted as the correlation length of the surface height in the substrate direction. In contrast, for the LC model the propagation distance is much larger than the correlation length. In general, the damage spreading distance (the propagation distance) is not always proportional to the correlation length. It is interesting that the survival probability of the damage decays with power law for the RSOS model, which is very similar to that of continuous absorbing transition [13].

We would like to thank B. Kahng for useful discussion, and S. U. Kim, K. Y. Moon, and D. Kim for numerical help. This work is supported in part by KOSEF (951-0206-005-2), the Hallym Academy of Sciences in Hallym University, the Ministry of Education (No. BSRI-95-2409), and KOSEF (No. 951-0206-003-2).

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